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HYPERSONIC FLOW PAST A DELTA WING AT LARGE ANGLES OF ATTACK*

V.N. GOLUBKIN

The thin shock layer method /1, 2/ is used to investigate the previously unknown mode of flow past a delta wing of finite span, at angles of attack close to $\pi/2$. The flow problem is formulated and analytic expressions are obtained for the gas-dynamic functions together with the equations expressing the relationships between the form of the wing surface and the shock wave. A method is given for solving inverse problems of flow past actual wings with an attached shock wave.

If the angle of attack remains finite, when the ratio ε of the densities on the shock wave tends to zero, the shock wave will remain attached to the sharp leading edge of the wing at any finite sweep-back angle. The basic results of the study of such a flow were given in /3/. On the other hand, when the angles of attack are close to $\pi/2$, a flow with a detached shock wave results.

Below it is shown that when $\varepsilon \ll 1$, a flow past a delta wing exists for the range of angles of attack close to $\pi/2$, $\alpha = \pi/2 - \varepsilon^{1/2}A$, with the shock wave attached to the wing tip, but attached to or detached from the leading edge, depending on the sweep-back angle. This mode falls between the two modes mentioned above, and lends itself to analytic study.

1. Let us consider hypersonic gas flow past a delta wing at large angles of attack

$$\alpha = \pi/2 - A_*, \quad 0 < A_* \ll 1 \tag{1.1}$$

Let $Oxyz$ be a Cartesian coordinate system attached to the wing (Fig.1). We assume that the thickness of the wing measured from the base plane $y = 0$ is small. Since the gas is strongly compressed in the leading shock wave, it follows that the shock surface will also be near the plane $y = 0$ and the small parameter of the thin shock layer method equal to the ratio of the densities across the shock will have the form

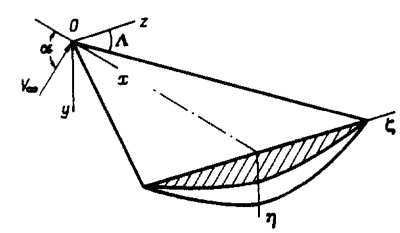

Fig.1

$$\varepsilon = \frac{\varkappa - 1}{\varkappa + 1}\left(1 + \frac{2}{m}\right) \tag{1.2}$$

$$m = (\varkappa - 1) M_\infty^2 = O(1)$$

where $\varkappa$ is the adiabatic index and M_∞ is the M number of the oncoming flow. We put $A_* = A\varepsilon^n$, $A = O(1)$ in (1.1).

We will obtain the order of magnitude of the perturbation by considering the flow past the leading edge of a plane wing with a finite sweep-back angle Λ ($\cos \Lambda = O(1)$). We will write the equation of the attached shock wave in the form

$$y_s = Y\,(x \cos \Lambda - z \sin \Lambda)$$

where $Y(\varepsilon, \alpha, \Lambda)$ is an unknown quantity, to be determined.

Using the well-known relations for the shock wave we obtain an expansion for the velocity component normal to the wing, which should vanish in accordance with the principle of impermeability. Taking into account the terms of lowest order of smallness we obtain, as $\varepsilon \to 0$,

$$\varepsilon^n AY \cos \Lambda - Y^2 - \varepsilon + \ldots = 0 \tag{1.3}$$

and this yields a solution corresponding to the weak branch of the shock wave

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$$Y = {}^1/_2\,[\varepsilon^n A \cos\Lambda - (\varepsilon^{2n} A^2 \cos^2\Lambda - 4\varepsilon)^{1/2}] \qquad (1.4)$$

We see that when $n < {}^1/_2$, and especially when $n = 0$ /3/, the shock is attached to the edge, for any finite sweep-back angle. When $n = {}^1/_2$, we must retain all terms in (1.3) and the expression under the square root may become negative. This implies that in the cross-section normal to the edge, the angle by which the flow rotates exceeds the limit value and the flow with attached shock wave does not materialize.

The case when $n = {}^1/_2$, $A \geqslant 2$, i.e.

$$\alpha = \pi/2 - \varepsilon^{1/2} A, \quad A \geqslant 2 \qquad (1.5)$$

is special. If we increase the sweep-back angle at a fixed angle at attack (1.5), then according to (1.4) we have at $\Lambda = \Lambda^* = \arccos(2/A)$ a transition from the mode of flow with a shock wave attached to the leading edge $(0 \leqslant \Lambda \leqslant \Lambda^*)$, to the mode with a shock wave detached from the edge $(\Lambda > \Lambda^*)$. At the same time, it is obvious /4, 5/ that the shock wave remains attached to the tip, i.e. a conical flow is possible. However, in a flow past a finite-size wing the influence of the trailing edge does not extend upstream if the flow is supersonic everywhere within the compressed layer. This condition will be formulated later. At large angles of attack ($n = {}^1/_2$, $A < 2$ or $n > {}^1/_2$) a flow with a shock wave attached to the edge is not possible. Further specification of the flow mode requires in this case (shock wave attached to the tip, or fully attached) a global solution of the problem of the streamline flow /6, 7/, or the use of experimental data /8/.

Let us consider the special case (1.5), which is intermediate between the flow past a wing with a shock wave attached to the edge for any value of Λ, and the flow with a shock wave detached from the edge for any Λ. Estimating the order of magnitude of the perturbed functions in the compressed layer in the range of values of the angle of attack under consideration we find, as $\varepsilon \to 0$, that

$$u \sim w \sim \varepsilon^{1/2} V_\infty, \quad v \sim \varepsilon V_\infty, \quad \rho \sim \varepsilon^{-1}\rho_\infty$$
$$\frac{p - p_\infty}{\rho_\infty V_\infty^2} - 1 \sim \varepsilon, \quad d \sim \varepsilon^{1/2} c \qquad (1.6)$$

where u, v, w are the velocity components along the x, y, z axes, p is the pressure, ρ is the density, d is the thickness of the compressed layer, c is the radical chord of the wing and the subscript ∞ denotes the parameters of the oncoming flow.

We see that since the flow arives at the wing almost head-on, the gas velocity in the compressed layer is low and comparable in order of magnitude with the speed of sound, while the thickness of the compressed layer has greater order of magnitude than that of the case when the angles of attack are finite /3/.

2. Let us now formulate and solve the problem. Using the estimates (1.6), we shall write the unknown functions in the form of expansions

$$u/V_\infty = \varepsilon^{1/2} u_0(\eta, \zeta) + \ldots, \quad v/V_\infty = \varepsilon v_1(\eta, \zeta) + \ldots \qquad (2.1)$$
$$w/V_\infty = \varepsilon^{1/2} w_0(\eta, \zeta) + \ldots, \quad p = p_\infty + \rho_\infty V_\infty^2\{1 + \varepsilon[p_1(\eta, \zeta) - A^2] + \ldots\}$$
$$\rho_\infty/\rho = \varepsilon - \varepsilon^2(1 - A^2 + p_1) - \varepsilon^2 m(m+2)^{-1}(u_0^2 + w_0^2) + \ldots$$
$$\left(\eta = \frac{y}{\varepsilon^{1/2} x}, \ \zeta = \frac{z}{x}\right)$$

Euler's equations for the conical flows /4/ and (2.1) together yield the following set of equations in the basic approximation of the thin shock layer method (the indices accompanying the functions are omitted):

$$(v - u\eta)\frac{\partial u}{\partial \eta} + (w - u\zeta)\frac{\partial u}{\partial \zeta} = 0 \qquad (2.2)$$

$$(v - u\eta)\frac{\partial w}{\partial \eta} + (w - u\zeta)\frac{\partial w}{\partial \zeta} = 0 \qquad (2.3)$$

$$\frac{\partial v}{\partial \eta} + \frac{\partial w}{\partial \zeta} - \left(\eta\frac{\partial u}{\partial \eta} + \zeta\frac{\partial u}{\partial \zeta}\right) = 0 \qquad (2.4)$$

$$(v - u\eta)\frac{\partial v}{\partial \eta} + (w - u\zeta)\frac{\partial v}{\partial \zeta} = -\frac{\partial p}{\partial \eta} \qquad (2.5)$$

$$(|\zeta| \leqslant Z, \ Z = \operatorname{ctg}\Lambda)$$

At the surface of the shock wave we have $\eta = \eta_s(\zeta)$, and the following conditions hold at the wing surface $\eta = \eta_b(\zeta)$:

$$u_s = A - \eta_s + \zeta\eta_s', \quad v_s = u_s(\eta_s - \zeta\eta_s') - 1 - \eta_s'^2 \qquad (2.6)$$
$$w_s = -\eta_s', \quad p_s = v_s + A(\eta_s - \zeta\eta_s') \quad (\eta_s' \equiv d\eta_s/d\zeta)$$
$$v_b = u_b\eta_b + (w_b - \zeta u_b)\eta_b' \quad (\eta_b' \equiv d\eta_b/d\zeta) \qquad (2.7)$$

We will solve the problem (2.2)–(2.5) as in /3, 9/, by introducing the function ψ', constant along the conic projection of the flow surface onto the plane $x = 1$, and equal to the coordinate ζ of the point where it intersects the shock wave

$$(v-u\eta)\frac{\partial\psi}{\partial\eta}+(w-u\zeta)\frac{\partial\psi}{\partial\zeta}=0,\quad \psi[\eta_s(\zeta),\zeta]=\zeta$$

Changing to new independent variables ζ, ψ , we obtain from (2.2), (2.3), (2.6)

$$u=u(\psi)=A-\eta_s(\psi)+\psi\eta_s'(\psi),\quad w=w(\psi)=-\eta_s'(\psi)$$

The inclination of the projection of the flow surfaces $\psi=\text{const}$ is equal to

$$\frac{\partial\eta}{\partial\zeta}=\frac{v-u\eta}{w-u\zeta} \tag{2.8}$$

Differentiating both sides of (2.8) with respect to ψ and using (2.4), we arrive at the equation

$$(w-u\zeta)\frac{\partial^2\eta}{\partial\zeta\,\partial\psi}+u(\psi)\frac{\partial\eta}{\partial\psi}=0$$

the solution of which, taking into account the condition at the wing $\eta=\eta_b(\zeta)$ when $\psi=\psi_b(\zeta)$, we shall write in the form

$$\eta(\psi,\zeta)=\eta_b(\zeta)+\int_{\psi_b}^{\psi}F(\xi)[w(\xi)-\zeta u(\xi)]\,d\xi \tag{2.9}$$

where the arbitrary functions F, ψ_b must be determined from the boundary conditions. To do this, we will obtain the function v from (2.8) and use conditions (2.6) in which $\eta_s(\zeta)=\eta(\zeta,\zeta)$. As a result, we have

$$F(\xi)=[w(\xi)-\xi u(\xi)]^{-2}$$

Then, satisfying conditions (2.7), we obtain the equation for determining the function $\psi_b(\zeta)$

$$F(\psi_b)\,\psi_b'(\zeta)\,[w(\psi_b)-\zeta u(\psi_b)]=0\quad (\psi_b'\equiv d\psi/d\zeta) \tag{2.10}$$

The above equation has two solutions, just as in the problem of flow past a wing with low aspect ratio /5, 9/. The first solution $\psi_b=\text{const}$ corresponds to the case when the wing surface coincides with the stream surface. The second solution corresponds to the stream function varying along the wing. The stream function is the inverse of the function $N=w/u$, i.e.

$$N[\psi_b(\zeta)]=\zeta \tag{2.11}$$

Satisfying conditions (2.10) ensures that the inclination of the projections of the flow surfaces onto the wing, and the inclination of the wing itself are the same, i.e. the wing surface is an envelope of the stream surfaces. The projection of the stream surface with a given value of ψ ends at the wing at the point with coordinate $\zeta=N(\psi)$.

The total velocity vector is directed at this point along the ray of the conical flow.

As a result we obtain from (2.8) the following expression for the velocity v :

$$\frac{v}{u(\psi)}=\eta-[N(\psi)-\zeta]\int_{\psi_b}^{\psi}\frac{d\xi}{[w(\xi)-\xi u(\xi)]^2}+[N(\psi)-\zeta]\,\eta_b'$$

Integrating (2.5) we obtain the pressure distribution

$$p=p_s(\zeta)+\int_{\psi}^{\zeta}[w(\psi_1)-\zeta u(\psi_1)]\frac{\partial v}{\partial\zeta}\frac{\partial\eta}{\partial\psi_1}\,d\psi_1$$

Thus, we have expressed all gas-dynamic parameters in terms of two functions describing the form of the wing, and the shock wave. In problems of streamlined flow only one of these functions needs to be specified, the other can be found from the solution. The equation connecting these functions directly can be obtained from (2.9)

$$\eta_s(\zeta)=\eta_b(\zeta)+\int_{\psi_b}^{\zeta}\frac{w(\xi)-\zeta u(\xi)}{[w(\xi)-\xi u(\xi)]^2}\,d\xi \tag{2.12}$$

Repeated differentiation taking (2.6), (2.10), (2.11) into account, enables us to write this relation in differential form

$$\eta_s''\left\{1-\frac{1+\zeta^2}{[\eta_s'(1+\zeta^2)+\zeta(A-\eta_s)]^2}\right\}=\eta_b''+\frac{\psi_b'(\zeta)}{u(\psi_b)[\psi_b(\zeta)-\zeta]^2} \tag{2.13}$$

$$\frac{\eta_s'(\psi_b)}{\eta_s(\psi_b)-A-\psi_b\eta_s'(\psi_b)}=\zeta \tag{2.14}$$

The above equations refer to the case of $\psi_b = \psi_b(\zeta)$. If on the other hand $\psi_b = \text{const}$, then the wing surface will coincide with the stream surface $\eta = \eta_t(\zeta)$. The form of the stream surfaces is described by (2.9) with $\psi_b = \text{const}$ and this, together with (2.12), yields

$$\eta_t'' = \eta_s''\left\{1 - \frac{1+\zeta^2}{[\eta_s'(1+\zeta^2)+\zeta(A-\eta_s)]^2}\right\} \tag{2.15}$$

The pressure distribution over the wing is given by

$$p_b(\zeta) = p_s(\zeta) + \eta_t''(\zeta)\int_{\psi_b}^{\zeta}\frac{[w(\xi)-\zeta u(\xi)]^3}{[w(\xi)-\xi u(\xi)]^2}d\xi \tag{2.16}$$

We note that the condition mentioned in Sect.1, stating that the flow is supersonic everywhere within the compressed layer has, by virtue of (2.1)–(2.3), (2.6), the form

$$(A - \eta_s + \zeta\eta_s')^2 + \eta_s'^2 > 1$$

and solutions for the infinite wing can also be used for wings with a finite chord.

3. The description of the wing surface in terms of the function $\psi = \psi_b(\zeta)$ and the system of equations (2.13), (2.14) are convenient for use in the case when the shock wave is attached to the edge. In the problem of a wing with detached shock wave the representation of the wing form as $\zeta = \zeta_b(\psi)$ was found to be more suitable. In this case the following condition analogous to (2.11) holds on the wing surface:

$$\zeta_b(\psi) = N(\psi) \tag{3.1}$$

and the equation describing the form of the shock wave in a flow past a plane wing becomes

$$N'(\psi)\eta_s''[N(\psi)]\left\{1 - \frac{1+N^2(\psi)}{[w[N(\psi)]-N(\psi)u[N(\psi)]]^2}\right\} = \frac{u(\psi)}{[w(\psi)-\psi u(\psi)]^3} \tag{3.2}$$

This shows that the curvature of the shock wave has a singularity at the point at which the expression within the curly brackets vanishes. Assuming, as in /5/, that the singularity lies on the characteristic straight line perpendicular to the wing and passing through the leading edge $\zeta = \zeta_b(\psi) = Z$ obtain, taking (3.1) into account, the mixed type boundary condition at the edge

$$(1+Z^2)\eta_s'(Z) + Z[A - \eta_s(Z)] = -(1+Z^2)^{1/2} \tag{3.3}$$

Expression (3.3) and symmetry condition $\eta_s'(0) = 0$ serve as the boundary conditions for (3.2). The solution of this problem shows that the conical modes of the streamlined flow with a shock wave attached to the tip only and with the flow choked at the leading edge (normal component of gas velocity equal to the speed of sound /6/) are also possible when $A < 2$.

4. Let us consider streamlined flow with a shock wave attached to the leading edge. The stream function cannot vary over the whole surface of the wing.

Let us assume the contrary. Then (2.11) holds on the whole wing surface and we have ($\zeta = Z$): $N = Z$ on the lading edge. This implies that the slope of the streamline behind the shock

$$\eta_t'|_s = -w_s - \frac{1}{w_s - \zeta u_s} \tag{4.1}$$

becomes infinite at this point. In this case, in order not to violate the condition of impermeability, we must assume that the inclination of the wing is also infinite. But then an attached shock wave cannot exist. Thus it follows that the stream function can vary only over that part of the wing surface which is coupled to the shock wave through the (stream) surface with constant $\psi_b = Z$.

In the case of a delta wing the shock wave attached to the edge is plane on a certain segment, and this corresponds to one of the solutions of (2.13) where $\eta_b = \eta_t = 0$, $\psi_b = \text{const}$

$$\eta_s' = \text{const}$$

We note that the equation has another solution describing a curved shock wave above a plane wing, which can be used when constructing a solution of the direct problem

$$\eta_s = A + \sqrt{1+\zeta^2}\,(\pm\operatorname{arctg}\zeta + \text{const})$$

Coupling the smooth flow behind the plane shock wave with the vortical flow behind the curved shock wave near the symmetry plane, will obviously lead to the appearance of a number of singularities in the form of discontinuities, just as in /2, 10/.

We shall use the solution of the inverse problem to illustrate the possiblity of constructing a wing in closed form, with a smooth attached shock wave. Since the problem is symmetrical, we shall consider the range $\zeta \geqslant 0$. Let us specify the shock wave in the form of a parabola

$$\eta_s = -\tfrac{1}{2}b\zeta^2, \quad b > 0 \tag{4.2}$$

From (2.14) we find

$$\psi_b(\zeta)=\frac{1}{\zeta}\left[\left(1+\frac{2A}{b}\zeta^2\right)^{1/2}-1\right] \tag{4.3}$$

The form of the wing segment on which the stream function varies when $0\leqslant\zeta\leqslant\zeta_*$, is obtained from (2.12) by means of quadratures, taking (4.2) and (4.3) into account. To couple this part to the shock wave when $\zeta_*\leqslant\zeta\leqslant Z$, we use one of the stream surfaces (2.15) approaching along the tangent to the wing surface enveloping them, of the form already determined. The magnitude of the sweep-back angle of the resulting wing $\Lambda = \operatorname{arcctg} Z$ and the coordinate ζ_* depend on the choice of one or other stream surface.

We can avoid the appearance of a cusp in the wing form by choosing a stream surface such that the motion along it, from the shock wave to the body, takes place in the direction towards the plane of symmetry $\zeta = 0$, which therefore corresponds to the flow-off line. This takes place when $\psi_b(\zeta)>\zeta$, which according to (4.3) holds when $b<A$, in the range $\zeta< a$ $(a^2=2(A/b-1))$. When $b>A$ the flow line lies in the plane of symmetry. A physical, real form of the wing obtains when the slope of the stream surface behind the shock wave is negative at the edge $\eta_s'(Z)<0$. This occurs when $b<A-\sqrt{2}$, $\zeta_1<Z<\zeta_2$ $(\zeta_{1,2}^2=a^2/2\mp(a^4/4-2/b^2)^{1/2})$. So-called critical /2, 9/ cross-sections ζ_c, may exist within the field of flow, in which the curvature of the projections of the stream surfaces is equal to zero and changes its sign on passing across these sections. The cross-sections correspond to the cases when the expressions within the curly brackets in (2.13) vanish. At the same time, since the centrifugal forces change their direction in the shock layer when $\zeta=\zeta_c$, the pressure difference $p_b(\zeta)-p_s(\zeta)$ changes its sign on the body and on the shock wave.

The pressure in the plane of symmetry is given in the form

$$p_s(0) = -1$$
$$p_b(0)=p_s(0)+\frac{b}{2(A-b)^4}\left[b^3-6Ab^2+3A^2b+2A^3+6A^2b\ln\frac{b}{A}\right]$$

Moreover, near the plane of symmetry $(\zeta\ll 1)$ the body has the form of a parabola,

$$\eta_b=\eta_{b0}-{}^1/_2B\zeta^2;$$
$$\eta_{b0}=-\frac{A-b+b\ln(b/A)}{(A-b)^2},\quad B=b+\frac{A^2}{2(A-b)^3}$$

but unlike the solution at finite angles of attack /9/ it is not equivalent to the form of the body (4.2) since $B>b$.

Another difference lies in the fact that the equations (2.13), (2.14) and other equations contain not only the derivatives η_s', η_s'', but also the function η_s itself. Therefore, writing the shock wave in the form

$$\eta_s=\eta_{s0}-{}^1/_2b\zeta^2$$

instead of (4.2), will change the solution, e.g.

$$p_s(0)=2A\eta_{s0}-\eta_{s0}^2-1$$
$$\eta_{b0}=\eta_{s0}-\frac{A-\eta_{s0}-b+b\ln b/(A-\eta_{s0})}{(A-\eta_{s0}-b)^2}$$

while all the remaining formulas beginning from (4.3) retain their form provided that A in them is replaced by $A-\eta_{s0}$.

Fig.2 shows the configuration of the shock wave, stream surfaces and the envelope for $A=3$, $b=1$. In accordance with what was said before, any stream surface intersecting the shock wave in the range $0.765<\zeta<1.848$, e.g. the surfaces 1—5, can be chosen as the console part of the wing. As a result we obtain a family of curves, where the thickness and prescribed form is that of the attached shock wave (4.2).

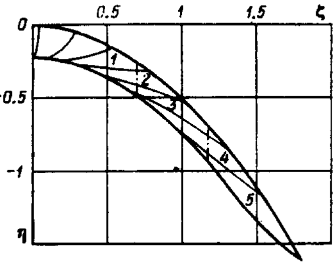

Fig.2

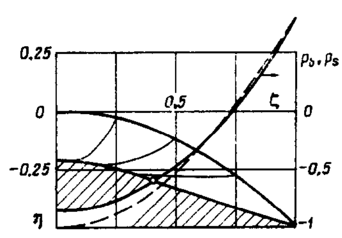

Fig.3

In the present case we have two critical cross-sections $\zeta_{c1} = 0.69, \zeta_{c2} = 1.17$ in the range $0 < \zeta < a$, shown in Fig.2 by the dot-dash lines.

The form of the wing surface with the sweep-back angle of $\Lambda = \pi/4$ is shown in Fig.3. A dot denotes the place where parts of the surface couple with the distributed and constant stream functions. Moreover, the figure shows the pressure distribution over the wing surface $p_b(\zeta)$, computed using (2.16) (solid line) and the pressure directly behind the shock wave $p_s(\zeta)$ (the dashed line).

Note that for certain specified configurations of the shock wave, in addition to the sufficient conditions for constructing a smooth wing surface, the equation $\psi_b(\zeta) = \zeta$ also holds at one or several points $\zeta \neq 0$, while additional lines of flow (flow-off) appear in the field of flow together with the line of flow (flow-off) in the plane of symmetry, just as in the case of flow at finite anlges of attack /9, 11/.

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A VARIATIONAL PRINCIPLE IN THE HYDROMECHANICS OF AN ISOTROPICALLY MAGNETIZABLE MEDIUM*

I.E. TARAPOV

The ideas expressed in /1/ are used as the basis for formulating a variational principle for describing the motion of an isotropically magnetizable medium. Representations are obtained for the velocity field, magnetic field and enthalpy written in terms of the Lagrange multipliers. New integrals of the equation of motion are derived.

The system of equations describing the non-relativistic motion of perfect magnetizable media can be written in the form /2/ (M is the magnetization of the medium)

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0, \quad \frac{dS'}{dt} = \frac{d}{dt}(S + S^*) = 0 \tag{1}$$

$$\operatorname{div} \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} - \operatorname{rot}[\mathbf{v}, \mathbf{B}] = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p - \nabla \psi^{(\rho)} + M \nabla H + \frac{1}{4\pi}[\operatorname{rot} \mathbf{H}, \mathbf{B}]$$

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